7.

Problem 11.58P (HRW)

(a) *Show that the rotational inertia of a solid cylinder of mass M radius R about its central axis is equal to the rotational inertia of a thin loop of mass M and radius* $R/\sqrt{2}$ *about its central axis.* (b) *Show that the rotational inertia I of any given body of mass M about any given axis is equal to the rotational inertia of an* ***equivalent loop*** *about that axis, if the loop has the same mass M and radius k given by*

$$k = \sqrt{\frac{I}{M}}.$$

The radius k of the equivalent loop is called the ***radius of gyration*** *of the given body.*

Solution:

(a)

In problem 1 we have calculated expressions for rotational inertia of different geometrical bodies including that of a cylinder of mass *M* and radius *R* about its central axis. We use the result that the rotational inertia *I* of a cylinder about its central axis is

$$I = \tfrac{1}{2}MR^2.$$

The rotational inertia of a thin loop of mass *M* and radius *k* about its central axis Mk^2 is basically the definition of rotational inertia. We thus have the relation

$$Mk^2 = \tfrac{1}{2}MR^2,$$

or

$$k = \frac{R}{\sqrt{2}}.$$

(b)

By definition the radius of gyration *k* for a body with moment of inertia *I* about a given axis is the radius of a loop of the same mass *M* and having the same moment of inertia as that of the body about the same axis passing through its centre. Therefore,

$$Mk^2 = I,$$

or

$$k = \sqrt{I/M}.$$