11.

Problem 11.85P (HRW)

A uniform helicopter rotor blade is 7.80 m *long and has a mass of* 110 kg. (a) *What force is exerted on the bolt attaching the blade to the rotor axle when the rotor is turning at* 320 rev min^{-1}? (*For this calculation, the blade can be considered a point mass at the centre of the mass. Why?*) (b) *Calculate the torque that must be applied to the rotor to bring it to full speed from rest in* 6.7 s. *Ignore air resistance.* (*The blade cannot be considered to be a point mass for this calculation. Why not? Assume the mass distribution of a uniform, thin rod.*)

(c) *How much work did the torque do on the blade in order for the blade to reach a speed of* 320 rev min^{-1}*?*

Solution:

(a)

If we assume that the mass distribution of helicopter blade is that of a thin uniform rod, its centre of mass will be at its midpoint. Length of the blade is 7.80 m.

Therefore, distance of its centre of mass from the bolt joining the blade to the rotor will be 3.9 m. As the blade is undergoing uniform circular motion the centripetal force on the blade will be $m\omega^2 r_{cm}$, where r_{cm} is the distance of the centre of mass of the blade from the rotor. Therefore, from the Newton's third law the force on the bolt joing the blade to the rotor will be $m\omega^2 r_{cm}$. Its magnitude is

$$=110\times\left(320\times 2\pi/60\right)^2\times 3.9\ \text{N}=4.8\times 10^5\ \text{N}.$$

(b)

Torque τ that must be applied to the rotor for bringing it to full speed in 6.7 s will be given by the relations

$$\omega=\alpha t,$$

$$\alpha=\frac{\tau}{I},$$

which gives

$$\tau=\frac{I\omega}{t}.$$

For calculating the torque blade is to be considered as a rigid body for the speed of revolution of a point on the blade at a distance r from the rotor is ωr. Its rotational inertia I is that of a rod of length l and mass m about one of its ends. It is $\frac{1}{3}ml^2$,

$$I = \frac{1}{3} \times 110 \times 7.8^2 \text{ kg m}^2 = 2230.8 \text{ kg m}^2.$$

This gives

$$\tau = 2230.8 \times (\frac{320 \times 2\pi}{60}) \times \frac{1}{6.7} \text{ N m} = 1.1 \times 10^4 \text{ N m}.$$

(c)

The work done by the torque in making the blade to reach angular speed ω from rest will be $\frac{1}{2} I\omega^2$. It is

$$W = \frac{1}{2} \times 2230.8 \times \left(\frac{320 \times 2\pi}{60} \right)^2 \text{ J} = 1.25 \times 10^6 \text{ J}.$$