14.

Problem 11.88P (HRW)

Attached to each end of a thin steel rod of length 1.20 m *and mass* 6.40 kg *is a small ball of mass* 1.06 kg. *The rod is constrained to rotate in a horizontal plane about a vertical axis through its midpoint. At a certain instant, it is observed to be rotating with an angular velocity of* 39.0 rev/s. *Because of friction, it comes to rest* 32.0 s *later. Assuming a constant frictional torque, we have to compute* (a) *the angular acceleration,* (b) *the retarding torque exerted by the friction,* (c) *the total mechanical energy dissipated by the friction, and* (d) *the number of revolutions exerted during the* 32.0 s. (e) *Now suppose that the frictional torque is known not to be constant. Which, if any, of the quantities* (a), (b), (c) *or* (d) *can still be computed without additional information? If such a quantity exits, we have to give its value.*

Solution:

Let l be the length of steel rod and M (6.40 kg) be its mass. Let m (1.06 kg) be the mass of balls attached to each end of the rod at a distance $l/2$ from the centre of the rod. The rotational inertia of the system about axis perpendicular to the rod and passing through its centre is

$$I = \frac{1}{12}Ml^2 + 2\times m(l/2)^2,$$

$$= (\frac{1}{12}\times 6.4 + \frac{1}{2}\times 1.06)1.20^2 \text{ kg m}^2,$$

$$= 1.53 \text{ kg m}^2.$$

At an instant when the rod is rotating with angular speed 39.0 rev/s its angular speed in rad s^{-1} is

$$\omega = 39\times 2\pi \text{ rad s}^{-1} = 245 \text{ rad s}^{-1}.$$

(a)

It is given that due to friction rod comes to a stop from the instant when its angular speed was 245 rad s^{-1} in 32 s. If we assume that the torque exerted by friction on the rod was constant, the acceleration of the rod during this period was

$$\alpha = -\frac{245}{32} \text{ rad s}^{-2} = -7.66 \text{ rad s}^{-2}.$$

(b)

Retarding torque exerted by friction on the system is given by

$\tau = I\alpha = -1.53 \times 7.66$ kg m^{2}s^{-2} $= 11.71$ kg m^{2}s^{-2}.

(c)

$$\text{Total energy dissipated by the system} = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} \times 1.53 \times \left(245\right)^2 \text{ J}$$

$$= 4.6 \times 10^4 \text{ J.}$$

(d)

Number of revolutions executed by the system during 32 s can be calculated from the relation

$\omega^2 = 2\alpha\theta.$

We calculate θ,

$$\theta = \frac{(245)^2}{2 \times 7.66} \text{ rad} = 3918 \text{ rad,}$$

$$= \frac{3918}{2\pi} \text{ rev} = 624 \text{ rev.}$$

(e)

When the frictional force is not known to be constant, from the given data, we can calculate only the amount of energy dissipated by the system.