Problem 16.44E (HRW)

A (hypothetical) large slingshot is stretched 1.50 m to launch a 130 g projectile with speed sufficient to escape from Earth (11.2 km/s). We can assume that the elastic sling shot obeys Hook's law. We have to find (a) the spring constant of the device, if all the elastic energy is converted to kinetic energy. (b) Assuming that an average person can exert a force of 220 N, we have to estimate the number of people required for stretching the slingshot.

Solution:

(a)

The spring constant, k, of the device can be determined by requiring that the potential energy of the slingshot will be $\frac{1}{2}k a^2$, as the slingshot obeys Hook's law and a is the maximum stretch. As the potential energy of the slingshot is fully converted into kinetic energy of the projectile, we have the condition

$$\frac{1}{2}ka^2 = \frac{1}{2}mv^2,$$

where v is the speed with which the projectile of mass m = 0.130 kg is released from the stretched slingshot. As the projectile is released with the escape velocity of the Earth, v = 11.2 km s⁻¹. We calculate the value of the spring constant

$$k = \frac{0.130 \times (11.2 \times 10^3)^2}{1.50^2} \text{ N m}^{-1} = 7.25 \times 10^6 \text{ N m}^{-1}.$$

Elastic force exerted on the slingshot in this situation is

$$F = 7.25 \times 10^6 \times 1.50 \text{ N} = 10.87 \times 10^6 \text{ N}.$$

(b)

It is given that an average person can exert a force of 220 N. Therefore, the number of persons needed to stretch the slingshot as per the data of the problem would be

$$N = \frac{10.87 \times 10^6}{220} = 49,432.$$