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Problem 19.29 (RHK)

We have to show (a) *that the intensity I is the product of the energy density u* (*energy per unit volume*) *and the speed of propagation v of a wave disturbance; that is I = uv*. (b) *We have to calculate the energy density in a sound wave* 4.82 km *from a* 47.5-kW *siren, assuming the waves to be spherical; the propagation is isotropic with no atmospheric absorption, and the speed of sound to be* 343 m/s.

Solution:

(a)

Let u be the energy density of the wave and v its speed of propagation. The intensity of the wave is defined as the amount of energy passing through a unit area perpendicular to the direction of propagation in one second. This will be the energy contained in volume v m^3. Therefore, the intensity I of the wave is given by

$$I = uv.$$

(b)

It is given that the waves emitted by the siren propagate spherically. The power of the siren is 47.5 kW. Therefore, as the propagation of the wave is isotropic and without absorption, the total energy flowing per second through a spherical surface of radius 4.82 km will be equal to the power of the siren. This condition determines the intensity of the wave at a distance of 4.82 km from the siren. We therefore have

$$4\pi\left(4.82\times10^{3}\text{ m}\right)^{2} I\left(4.82\text{ km}\right)=47.5\times10^{3}\text{ W},$$

or

$$I\left(4.82\text{ km}\right)=\frac{47.5\times10^{3}}{4\pi\left(4.82\times10^{3}\text{ m}\right)^{2}}\text{ W m}^{-2}=0.163\text{ mW m}^{-2}.$$

(c)

The speed of the sound wave is 343 m s^{-1}. Therefore, from the definition of the intensity we calculate the energy density u,

$$u = \frac{I}{v} = \frac{0.163 \times 10^{-3}}{343} \text{ J m}^{-3},$$

$$= 474 \times 10^{-9} \text{ J m}^{-3}.$$