150.

Problem 20.35 (RHK)

A large parabolic reflector having a circular opening of radius 0.50 m *is used to focus sound. If the energy is delivered from the focus to the ear of a listening detective through a tube of diameter* 1.0 cm *with* 12% *efficiency, we have to find the distance away from which a whispered conversation can be understood. We can assume that the sound level of a whisper is* 20 dB *at* 1.0 m *from the source, considered to be a point, and that the threshold for hearing is* 0 dB.

Solution:

We are given that the sound level of a whisper at 1.0 m from the source is 20 dB. From this information we estimate the intensity of sound of a whisper at a distance of 1.0 m from its source.

The definition of *SL* is

$$SL = 10\log\left(\frac{I}{I_0}\right),$$

where

$$I_0 = 10^{-12}\ \text{W m}^{-2}.$$

So the magnitude of I is

$$I = 10^2 \times 10^{-12}\ \text{W m}^{-2} = 10^{-10}\ \text{W m}^{-2}.$$

Therefore, the intensity of whisper sound at a distance R (m) from its source will be

$$I(R) = \frac{I(1\ \text{m})}{R^2} = \frac{10^{-10}}{R^2}\ \text{W m}^{-2}.$$

The amount of energy per second picked up by the parabolic reflector of radius 0.5 m will be

$$w = \frac{10^{-10} \times \pi \times (0.5)^2}{R^2}\ \text{W}.$$

We are given that sound from the focus is picked up by a tube of 1.0 cm diameter with an efficiency of 12%. Therefore, the intensity of sound received by the ear of the listener will be

$$I_{ear} = \frac{w \times 0.12}{\pi\left(0.5 \times 10^{-2}\right)^2}\ \text{W m}^{-2} = \frac{0.12 \times 10^{-10}}{10^{-4} \times R^2}\ \text{W m}^{-2},$$

$$= \frac{12 \times 10^{-8}}{R^2}\ \text{W m}^{-2}.$$

If the sound is to be just audible, the condition is that its *SL* has to be 0 dB. This condition implies

$$\frac{12\times10^{-8}}{R^2}\ \text{W m}^{-2} = 10^{-12}\ \text{W m}^{-2},$$

or

$$R^2 = 12\times10^4,$$

and

$$R = 3.46\times10^2\ \text{m} = 346\ \text{m}.$$