280.

Problem 27.21 (RHK)

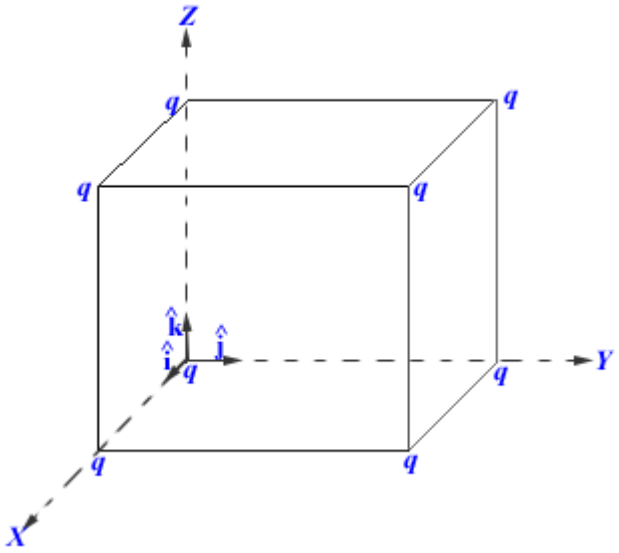
A cube of edge a carries a point charge q at each corner. We have to show that the resultant electric force on any one of the charges is given by

$$F = \frac{0.262\, q^2}{\varepsilon_0 a^2} \ ,$$

directed along the body diagonal away from the cube.

Solution:

In the diagram a cube has been drawn with length *a* of each edge. We have placed a charge of q **C** at each eight vertices of the cube.



A set of Cartesian co-ordinate axes have been laid out with origin at one of the vertices of the cube as shown in the diagram. Unit

vectors along the ***X***,***Y,*** and **Z** axes are $\hat{i}$, $\hat{j}$, and $\hat{k}$, respectively. We calculate the force on the charge q at the vertex of the cube, which is at the origin of the co-ordinate system. By symmetry force on any other charge due to the other 7 charges and in a direction relative to them will be the same.

By Coulomb's law

$$\vec{F} = \frac{q^2}{4\pi\varepsilon_0 a^2}\left(\begin{array}{l} -\hat{i}-\hat{j}-\hat{k}-\left(\frac{\hat{i}+\hat{j}}{2\sqrt{2}}+\frac{\hat{j}+\hat{k}}{2\sqrt{2}}+\frac{\hat{k}+\hat{i}}{2\sqrt{2}}\right) \\ -\frac{1}{3\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right) \end{array}\right).$$

Or

$$\vec{F} = -\frac{q^2}{4\pi\varepsilon_0 a^2}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right)\times\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$= -\frac{q^2}{\varepsilon_0 a^2}\left(\frac{1}{4\pi}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right)\right)\times\left(\hat{i}+\hat{j}+\hat{k}\right).$$

We note that the magnitude of $\vec{F}$ is

$$\left|\vec{F}\right| = \frac{q^2}{\varepsilon_0 a^2}\times\frac{1}{4\pi}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right)\times\sqrt{3}.$$

As

$$\frac{1}{4\pi}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right)\times\sqrt{3} = 0.262\ ,$$

$$\left|\vec{F}\right| = \frac{0.262\,q^2}{\varepsilon_0 a^2}.$$

And the direction of the force on the charge due to the other 7 charges q at the other vertices of the cube will be $\frac{\vec{F}}{\left|\vec{F}\right|} = -\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}}$. That is it is directed along the body diagonal away from the cube.