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Problem 30.20 (RHK)

The electric field inside a nonconducting sphere of radius R, containing uniform charge density, is radially directed and has magnitude

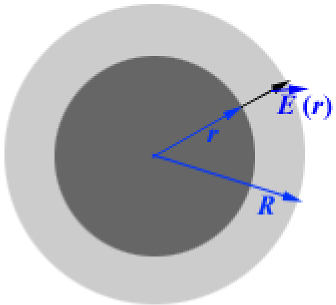
$$E(r) = \frac{qr}{4\pi\varepsilon_0 R^3},$$

Where q is the total charge in the sphere and r is the distance from the centre of the sphere. (a) *We have to find the potential* $V(r)$ *inside the sphere, taking* $V = 0$ *at* $r = 0$. (b) *We have to find the difference in potential between a point on the surface and the centre of the sphere. If q is positive, we have to answer which point is at higher potential.* (c) *We have to show that the potential at a distance r from the centre, where* $r < R$, *is given by*

$$V = \frac{q\left(3R^2 - r^2\right)}{8\pi\varepsilon_0 R^3}$$

Where the zero of the potential is taken at $r = \infty$. *We have to explain why this result differs from that of the part* (a).

Solution:



We are given a nonconducting sphere containing uniform charge density. Let q be the total charge contained inside the sphere of radius R. The volume charge density will be

$$\rho = \frac{q}{4\pi R^3/3} = \frac{3q}{4\pi R^3}.$$

The electric field at a distance r from the centre $r < R$ will be radially directed as shown in the figure and will be proportional to the total charge contained inside the sphere of radius r. By applying gauss form of the Coulomb' law, we get

$$E(r)=\frac{1}{4\pi\varepsilon_0}\frac{\left(4\pi r^3/3\right)\rho}{r^2}=\frac{1}{4\pi\varepsilon_0}\times\frac{\left(4\pi r^3/3\right)}{r^2}\times\frac{3q}{4\pi R^3}$$

$$=\frac{qr}{4\pi\varepsilon_0 R^3}.$$

(a)

Potential $V(r)$ inside the sphere, taking $V(r=0)=0$ can be calculated from the definition of potential difference in electric field. It will be

$$V(r)=-\int_0^r E(r)dr=-\int_0^r \frac{qr}{4\pi\varepsilon_0 R^3}dr=-\frac{qr^2}{8\pi\varepsilon_0 R^3}. \quad \text{(A)}$$

(b)

Therefore, the difference in potential between a point on the surface of the sphere of radius R and its centre will be

$$V(R)=-\int_0^R E(r)dr=-\frac{q}{8\pi\varepsilon_0 R}\ .$$

If $q>0$, the centre of the sphere will be at higher potential than at its outer surface.

(c)

We will next calculate the potential $V(r)$ at $r<R$, by fixing that $V(r=\infty)=0$.

Note that electric field for $r>R$ is

$$E(r) = \frac{q}{4\pi\varepsilon_0 r^2} \quad .$$

Therefore,

$$V(r) - V(\infty) = -\int_{\infty}^{r} \vec{E}(r).d\vec{r} = -\int_{\infty}^{R} \frac{q}{4\pi\varepsilon_0 r^2} dr - \int_{R}^{r} \frac{qr}{4\pi\varepsilon_0 R^3} dx$$

$$= \frac{q}{4\pi\varepsilon_0 R} - \frac{q\left(r^2 - R^2\right)}{8\pi\varepsilon_0 R^3}$$

$$= \frac{q\left(3R^2 - r^2\right)}{8\pi\varepsilon_0 R^3} \quad . \qquad \text{(B)}$$

The results (A) and (B) differ because in (B) we have taken potential $V(\infty) = 0$.

Potential at $r = \infty$ under the condition of (A) will be

$$V(\infty) = V(R) + \left(V(\infty) - V(R)\right)$$

$$= -\frac{q}{8\pi\varepsilon_0 R} - \int_{R}^{\infty} \frac{q}{4\pi\varepsilon_0 r^2} dr = -\frac{3q}{8\pi\varepsilon_0 R}.$$

If we want to fix $V(\infty) = 0$, then from the result of (A) we should subtract $V(\infty)$. We have

$$V(r) = -\frac{qr^2}{8\pi\varepsilon_0 R^3} + \frac{3q}{8\pi\varepsilon_0 R} = \frac{q\left(3R^2 - r^2\right)}{8\pi\varepsilon_0 R^3} \quad .$$