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Problem 30.19 (RHK)

A Geiger counter has a metal cylinder 2.10 cm *in diameter along whose axis is stretched a wire* 1.34×10^{-4} cm *in diameter. A potential difference of* $855\ \text{V}$ is *applied between them. We have to find* (a) *the electric field at the surface of the wire;* (b) *at the surface of the cylinder.*

Solution:

We will use Gauss' law to find the expression for the electric field inside the Geiger counter. Let σ be the charge per unit length on the inner wire. Because of cylindrical symmetry electric field will be perpendicular to the axis of the cylinder. We consider a cylindrical Gaussian surface of radius r centred at the axis of the Geiger counter. By applying Gauss' law, we have

$$2\pi r\varepsilon_0 E(r) = \sigma,$$

or

$$E(r) = \frac{\sigma}{2\pi\varepsilon_0 r} .$$

Let r_a be the radius of the wire $\left(0.67\times10^{-4}\text{ cm}\right)$ and r_b (1.05 cm) be the radius of the metallic cylinder of the Geiger counter. The potential difference across the Geiger counter will be

$$V = V(r_b) - V(r_a) = -\int_{r_a}^{r_b} \vec{E}(r).d\vec{r} = \int_{r_a}^{r_b} \frac{\sigma}{2\pi\varepsilon_0 r} dr$$

$$= \frac{\sigma}{2\pi\varepsilon_0} \ln\left(\frac{r_b}{r_a}\right) .$$

Therefore,

$$E(r) = \frac{1}{r} \times \frac{V}{\ln\left(\frac{r_b}{r_a}\right)} .$$

Electric field at the surface of the wire will be

$$E(r_a) = \frac{1}{r_a} \times \frac{V}{\ln\left(\frac{r_b}{r_a}\right)} = \frac{855}{0.67\times10^{-6}\ln\left(\frac{1.05}{0.67\times10^{-4}}\right)} \text{ V m}^{-1}$$

$$= 132 \text{ MV m}^{-1}.$$

Electric field at the metallic cylinder of the Geiger counter will be

$$E(r_b) = \frac{1}{r_b} \times \frac{V}{\ln\left(\frac{r_b}{r_a}\right)} = \frac{855}{1.05\times10^{-2}\ln\left(\frac{1.05}{0.67\times10^{-4}}\right)} \text{ V m}^{-1}$$

$$= 8.429 \text{ kV m}^{-1}.$$