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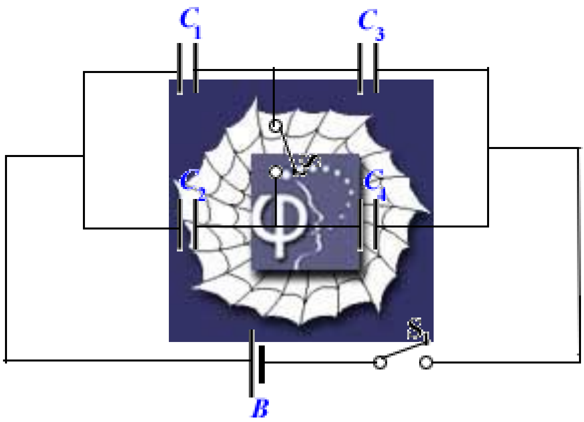
Problem 31.27 (RHK)

In the circuit shown in the figure, the battery B *supplies* 12 V. *We have to find* (a) *the charge on each capacitor when switch* S_1 *is closed and* (b) *when* (*later*) *switch* S_2 *is also closed. We make take* $C_1 = 1.0\ \mu\text{F}$, $C_2 = 2.0\ \mu\text{F}$, $C_3 = 3.0\ \mu\text{F}$, *and* $C_4 = 4.0\ \mu\text{F}$.



Solution:

Data of the problem are

$C_1 = 1.0\ \mu\text{F}$,

$C_2 = 2.0\ \mu\text{F}$,

$C_3 = 3.0\ \mu\text{F}$,

$C_4 = 4.0\ \mu\text{F}$,

and the EMF of the battery B is $V = 12$ V.

(a)

Let the switch S_1 be closed first. Let the magnitude of charge on capacitors C_1 and C_3 be q_a μC. As these capacitors are connected in series, potential difference across them will add to 12 V. That is

$$q_a\left(\frac{1}{C_1}+\frac{1}{C_3}\right)=12,$$

and

$$q_a=\frac{12\,C_1C_3}{C_1+C_3}=\frac{12\times1\times3}{1+3}\ \mu\text{C}=9\ \mu\text{C}.$$

And let the charge across capacitors C_2 and C_4 be q_a' μC. Then

$$q_a'=\frac{12\,C_2C_4}{C_2+C_4}=\frac{12\times2\times4}{2+4}\ \mu\text{C}=16\ \mu\text{C}.$$

(b)

When the switch S_2 is also closed, the circuit has effectively two capacitors in series one is parallel combination of C_1 and C_2 with resultant capacitance C_1+C_2, the other is parallel combination of C_3 and C_4 with resultant capacitance C_3+C_4. Therefore, the effective capacitance in the circuit will be

$$C=\frac{(C_1+C_2)(C_3+C_4)}{C_1+C_2+C_3+C_4}=\frac{(1+2)(3+4)}{1+2+3+4}\ \mu\text{F}=\frac{21}{10}\ \mu\text{F}.$$

The combined charge on the capacitors C_1 and C_2, or the equal combined charge on the capacitors C_3 and C_4 will be

$$q=12\times\frac{21}{10}\ \mu\text{F}=\frac{126}{5}\ \mu\text{F}=25.2\ \mu\text{F}.$$

Let the charge on C_1 be q_1 μF. We have

$$\frac{q_1}{C_1}=\frac{q-q_1}{C_2},$$

or

$$q_1=\frac{qC_1}{C_1+C_2}=\frac{126}{5\times 3}\ \mu\text{F}=8.4\ \mu\text{F}$$

and

$$q_2=q-q_1=(25.2-8.4)\ \mu\text{F}=16.8\ \mu\text{F}.$$

Similarly, let the charge on C_3 be q_3 μF. We have

$$q_3=\frac{qC_3}{C_3+C_4}=\frac{25.2\times 3}{3+4}\ \mu\text{F}=10.8\mu\text{F},$$

and

$$q_4=q-q_3=(25.2-10.8)\ \mu\text{F}=14.4\ \mu\text{F}.$$