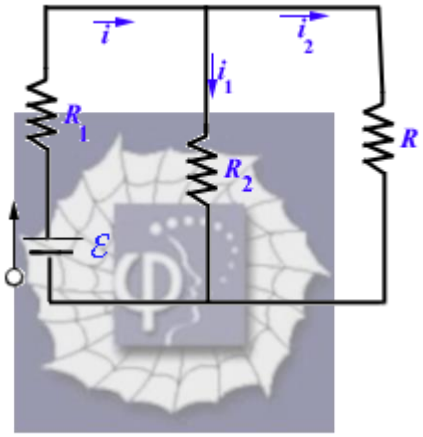
408.

Problem 33.26 (RHK)

In the circuit shown in the figure E, R_1, *and* R_2 *have constant values but R can be varied. We have to find an expression for R that results in the maximum heating in that resistor.*



Solution:

We will find the current flowing through R. We have a multi-loop circuit with one junction. Equations that follow from Kirchoff's laws are

$i = i_1 + i_2,$

$$i = \frac{\mathrm{E}}{R_1 + \dfrac{RR_2}{R + R_2}} = \frac{\mathrm{E}\left(R + R_2\right)}{R_1R_2 + R\left(R_1 + R_2\right)}.$$

Also,

$$i_2 R = i_1 R_2$$
$$= \left(i - i_2\right) R_2,$$

or

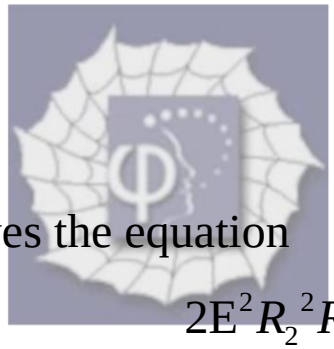
$$i_2 = \frac{iR_2}{R + R_2} = \frac{\mathrm{E}R_2}{R_1 R_2 + R\left(R_1 + R_2\right)}.$$

Rate of Joule heating in the resistance R is

$$P = {i_2}^2 R = \left(\frac{\mathrm{E}R_2}{R_1 R_2 + R\left(R_1 + R_2\right)} \right)^2 R.$$

We find the extremum solution of $P\left(R\right)$ by requiring

$$\left. \frac{dP\left(R\right)}{dR} \right|_{R=R_{\max}} = 0.$$



This condition gives the equation

$$\frac{\mathrm{E}^2 {R_2}^2}{\left(R_1 R_2 + R_{\max}\left(R_1 + R_2\right)\right)^2} - \frac{2\mathrm{E}^2 {R_2}^2 R_{\max}\left(R_1 + R_2\right)}{\left(R_1 R_2 + R_{\max}\left(R_1 + R_2\right)\right)^3} = 0.$$

As

$$R_1 R_2 + R_{\max}\left(R_1 + R_2\right) \neq 0,$$

we have

$$R_1 R_2 + R_{\max}\left(R_1 + R_2\right) - 2R_{\max}\left(R_1 + R_2\right) = 0,$$

or

$$R_{\max} = \frac{R_1 R_2}{R_1 + R_2}.$$