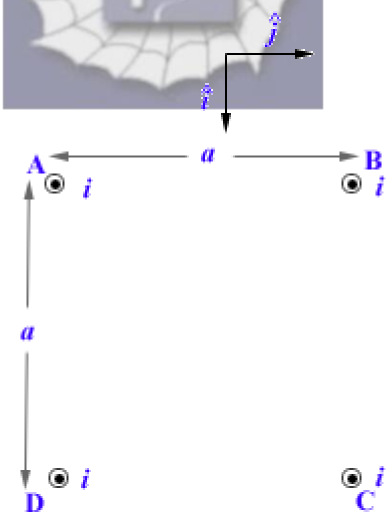
470.

Problem 35.35 (RHK)

Four long copper wires are parallel to each other and arranged in a square, as shown in the figure. They carry equal currents i *out of the page, as shown. We have to calculate the force per meter on any one wire. We have to give its magnitude and direction. We may assume that* $i = 18.7\ \text{A}$ *and* $a = 24.5\ \text{cm}$. (*In the case of parallel motion of charged particles in a plasma, this is known as the pinch effect.*)



Solution:

It is given that four long copper wires are parallel to each other and are arranged in a square. They carry equal currents i out of the page. We have to calculate the force per unit meter on any one of the wires. Current in the wires,

$i = 18.7\ \text{A}$,

and the separation between the wires,

$a = 24.5\ \text{cm}$.

As the wires are parallel and current is flowing in the same direction in each one of them, the force between any pair will be attractive and will act along the perpendicular line joining the wires. In the figure we have labelled the four wires as A, B, C and D. Let us compute the force per unit length on the wire C.

We will derive the formula for the force between two current carrying parallel wires by computing the force on C due to B.

We fix the coordinate system by drawing unit vectors $\hat{i}$ and $\hat{j}$, as shown in the figure.

$\hat{i} \times \hat{j} = \hat{k}$.

The magnetic field at C due to B will be

$$\vec{B} = \frac{\mu_0 i}{2\pi a}\hat{j}.$$

According to the Lorentz force law, the force on a segment $d\vec{s}$ carrying current i in a magnetic field $\vec{B}$ is given by

$$d\vec{F} = i d\vec{s} \times \vec{B}.$$

As the current in C is in the direction $\hat{k}$, the force per unit length on C due to B will be

$$\vec{F}_{\mathrm{BC}} = i\hat{k} \times \frac{\mu_0 i}{2\pi a}\hat{j} = -\frac{\mu_0 i^2}{2\pi a}\hat{i}.$$

Similarly, the force per unit length on C due to D will be

$$\vec{F}_{\mathrm{DC}} = i\hat{k} \times \frac{\mu_0 i}{2\pi a}\left(-\hat{i}\right) = -\frac{\mu_0 i^2}{2\pi a}\hat{j}.$$

The magnetic field at C due to A is

$$\vec{B}_{\mathrm{AC}} = \frac{\mu_0 i}{2\pi\sqrt{2}a}\left(-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right).$$

Therefore, force per unit length on C due to A will be

$$\vec{F}_{\mathrm{AC}} = i\hat{k} \times \frac{\mu_0 i}{2\pi\sqrt{2}a}\left(-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) = \frac{\mu_0 i^2}{4\pi a}\left(-\hat{i} - \hat{j}\right).$$

Therefore, the force per unit length on wire C due to wires A, B and D will be

$$\vec{F}_{\mathrm{C}} = \vec{F}_{\mathrm{BC}} + \vec{F}_{\mathrm{DC}} + \vec{F}_{\mathrm{AC}} = -\frac{\mu_0 i^2}{2\pi a}\left(\hat{i} + \hat{j} + \frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

$$= -\frac{3\mu_0 i^2}{4\pi a}\left(\hat{i} + \hat{j}\right) = \frac{3\mu_0 i^2}{2\sqrt{2}\pi a}\left(-\frac{1}{\sqrt{2}}\hat{i} - -\frac{1}{\sqrt{2}}\hat{j}\right).$$

$-\frac{1}{\sqrt{2}}\hat{i} - -\frac{1}{\sqrt{2}}\hat{j}$ is a unit vector pointing toward the centre of the square from C.

We now put in the data and calculate the magnitude of the force per unit on any one of the wires because of the other three.

$$\left|\vec{F}_{\mathrm{C}}\right| = \frac{6}{\sqrt{2}}\frac{\mu_0}{4\pi}\frac{i^2}{a} = \frac{6\times 10^{-7}\times 18.7^2}{\sqrt{2}\times 24.5\times 10^{-2}}\ \mathrm{N} = 60.5\times 10^{-5}\ \mathrm{N}$$

$$= 606\ \mu\mathrm{N}.$$

The direction is toward the centre of the square. From symmetry, we conclude that force per unit length on any wire due to the other three will be of $606\ \mu\mathrm{N}$ and will be pointing toward the centre of the square formed by the four current carrying wires.