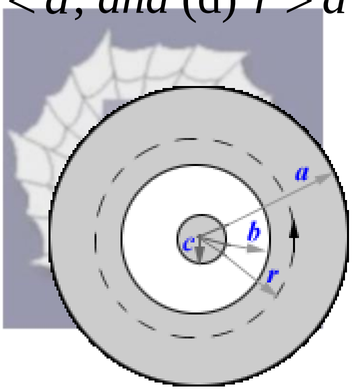
475.

Problem 35.45 (RHK)

In the figure a cross-section has been shown of a long conductor of a type called a coaxial cable of radii a, b, and c. Equal but antiparallel uniformly distributed currents i exist in the two conductors. We have to derive expressions for $B(r)$ *in the ranges* (a) $r < c$, (b) $c < r < b$, (c) $b < r < a$, *and* (d) $r > a$.



Solution:

In the figure a cross-section has been shown of a long conductor of a type called a coaxial cable of radii *a*, *b*, and *c*. Equal but antiparallel uniformly distributed currents *i* exist in the two conductors.

We assume that the current in the conductor of inner radius b and outer radius a is i and it is in the direction of the outward perpendicular to the page. The current in the conductor of radius c will also be i, but its direction will that be of the inward perpendicular to the page.

We will take appropriate Amperian circular loops for finding the magnetic field $B(r)$ using the Ampere's law.

(a)

For $r < c$,

$$\oint \vec{B}(r).d\vec{s} = \mu_0 \times (\text{current enclosed}),$$

or

$$2\pi r B(r) = \mu_0 i \frac{\pi r^2}{\pi c^2},$$

$$B(r) = \frac{\mu_0 i r}{2\pi c^2}. \quad (r < c)$$

(b)

For $c < r < b$,

$$\oint \vec{B}(r).d\vec{s} = \mu_0 \times (\text{current enclosed}),$$

or

$$2\pi r B(r) = \mu_0 i,$$

$$B(r) = \frac{\mu_0 i}{2\pi r}. \quad (c < r < b)$$

(c)

For $b < r < a$,

$$\oint \vec{B}(r) \cdot d\vec{s} = \mu_0 \times (\text{current enclosed}),$$

or

$$2\pi r B(r) = \mu_0 \left(i - \frac{i\pi\left(r^2 - b^2\right)}{\pi\left(a^2 - b^2\right)} \right),$$

$$\left|\vec{B}(r)\right| = \frac{\mu_0 i}{2\pi r} \frac{\left(a^2 - r^2\right)}{\left(a^2 - b^2\right)}. \quad (b < r < a)$$

(d)

For $r > a$,

The Amperian loop encloses two currents of equal magnitude but opposite in signs as the currents flow in opposite directions. Therefore, contributions to the right-hand side of the Ampere's law cancel each other. Therefore,

$$B(r) = 0.$$