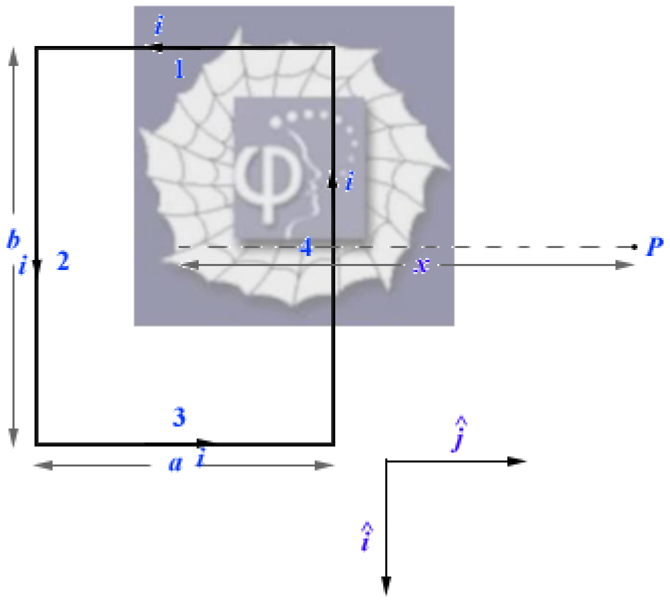
482.

<u>Problem 35.33 (RHK)</u>

Consider the rectangular loop carrying current i as shown in the figure. Point P is located a distance x from the centre of the loop. We have to find an expression for the magnetic field at P due to the current loop, assuming that P is very far away.



<u>Solution:</u>

We will calculate the magnetic field at P due to opposite sides of the rectangular taken in pairs. We first consider the sides 1 and 3. In the coordinate system fixed as shown, let the coordinates of the point P be $(b/2, a/2, x)$.

We assume that $x \gg a, b$, and the distance to point P from the sides of the loop can be approximated as nearly equal to x.

From the Ampere's law we note that the magnetic field at P due to line element $id\xi\left(-\hat{j}\right)$ at $\left(0,\xi,0\right)$ will be

$$d\vec{B} \approx \frac{\mu_0}{4\pi}\frac{i\left(-\hat{j}d\xi\right)\times\left(b\hat{i}/2+\left(a/2-\xi\right)\hat{j}+x\hat{k}\right)}{x^3}$$

$$= -\frac{\mu_0 i\left(-b\hat{k}/2+x\hat{i}\right)}{4\pi x^3}.$$

We note that the $\hat{i}$ components of $d\vec{B}$ due to current elements in sides 1 and 3 will cancel each other in pairs. Therefore, the component of $d\vec{B}$ in the $\hat{k}$ direction at P due to current i in the sides 1 and 3 of the loop will be

$$d\vec{B} = \frac{\mu_0 ibd\xi\,\hat{k}}{4\pi x^3},$$

and the combined field at P due to current flows in sides 1 and 3 of the loop will be

$$\vec{B}_{(1,3)} = \frac{\mu_0 iba\,\hat{k}}{4\pi x^3}.$$

We can calculate the magnetic field at P due to the current flow in the sides 2 and 4 of the loop and we will find that

$$\vec{B}_{(2,4)} = \frac{\mu_0 iba\,\hat{k}}{4\pi x^3}.$$

Therefore, the magnetic field at P due to current flow in the loop will be

$$\vec{B} = \frac{\mu_0 iba\,\hat{k}}{2\pi x^3}.$$

The magnetic dipole moment of the planar loop of area ab with current i is

$$\mu = abi.$$

Therefore,

$$\vec{B} = \frac{\mu_0 \mu\,\hat{k}}{2\pi x^3}.$$

This is the expression of field due to a magnetic dipole at a distance x on its axis far away from the dipole.