

555.

Problem 41.24 (RHK)

Frank D. Drake, an active investigator in SETI (Search for Extra-Terrestrial Intelligence) programme, has said that the large radio telescope in Arecibo, Puerto Rico, “can detect a signal which lays down on the entire surface of the Earth a power of only one picowatt”. We have to find (a) the power actually received by the Arecibo antenna for such a signal. The antenna diameter is 305 m. (b) We have to find the power output of a source at the centre of our galaxy that could provide such a signal. The galactic centre is 2.3×10^4 ly away. We can take the source as radiating uniformly in all directions.



Solution:

(a)

Radius of the Earth, $R_E = 6.37 \times 10^6$ m.

The surface of the Earth,

$$A = 4\pi R_E^2 = 4\pi \times (6.37 \times 10^6)^2 \text{ m}^2 = 509.9 \times 10^{12} \text{ m}^2.$$

The intensity of the weakest signal that can be detected by the Arecibo radio telescope will therefore be

$$I = \frac{1 \times 10^{-12}}{509.9 \times 10^{12}} \text{ W m}^{-2} = 1.96 \times 10^{-27} \text{ W m}^{-2}.$$

Diameter of the Arecibo antenna $d = 305 \text{ m}$.

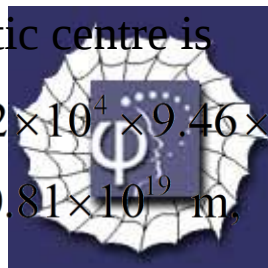
Therefore, the power received by the Arecibo radio telescope for the weakest signal will be

$$P = 1.96 \times 10^{-27} \times \pi \times (d/2)^2 = 1.96 \times 10^{-27} \times \pi \times (305/2)^2 \text{ W} \\ = 1.43 \times 10^{-22} \text{ W}.$$

(b)

Distance of the galactic centre is

$$D = 2.2 \times 10^4 \text{ ly} = 2.2 \times 10^4 \times 9.46 \times 10^{15} \text{ m} \\ = 20.81 \times 10^{19} \text{ m}.$$



where we have used that

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}.$$

Therefore, the power of the source emitting the signal from the centre of our galaxy will be

$$P_s = 1.96 \times 10^{-27} \times 4\pi \times (20.81 \times 10^{19})^2 \text{ W} \\ = 1.067 \times 10^{15} \text{ W} ; 1.1 \times 10^{15} \text{ W}.$$