555.

Problem 41.24 (RHK)

Frank D. Drake, an active investigator in SETI (*Search for Extra-Terrestrial Intelligence*) *programme, has said that the large radio telescope in Arecibo, Puerto Rico,* "*can detect a signal which lays down on the entire surface of the Earth a power of only one picowatt*". *We have to find* (a) *the power actually received by the Arecibo antenna for such a signal. The antenna diameter is* 305 m. (b) *We have to find the power output of a source at the centre of our galaxy that could provide such a signal. The galactic centre is* 2.3×10^4 ly *away. We can take the source as radiating uniformly in all directions.*

Solution:

(a)

Radius of the Earth, $R_E = 6.37\times10^6$ m.

The surface of the Earth,

$$A = 4\pi R_E^2 = 4\pi\times\left(6.37\times10^6\right)^2 \text{ m}^2 = 509.9\times10^{12} \text{ m}^2.$$

The intensity of the weakest signal that can be detected by the Arecibo radio telescope will therefore be

$$I = \frac{1\times10^{-12}}{509.9\times10^{12}}\ \text{W m}^{-2} = 1.96\times10^{-27}\ \text{W m}^{-2}.$$

Diameter of the Arecibo antenna $d = 305$ m.

Therefore, the power received by the Arecibo radio telescope for the weakest signal will be

$$P = 1.96\times10^{-27}\times\pi\times(d/2)^2 = 1.96\times10^{-27}\times\pi\times(305/2)^2\ \text{W}$$
$$= 1.43\times10^{-22}\ \text{W}.$$

(b)

Distance of the galactic centre is

$$D = 2.2\times10^{4}\ \text{ly} = 2.2\times10^{4}\times9.46\times10^{15}\ \text{m}$$
$$= 20.81\times10^{19}\ \text{m},$$

where we have used that

$$1\ \text{ly} = 9.46\times10^{15}\ \text{m}.$$

Therefore, the power of the source emitting the signal from the centre of our galaxy will be

$$P_S = 1.96\times10^{-27}\times4\pi\times\left(20.81\times10^{19}\right)^2\ \text{W}$$
$$= 1.067\times10^{15}\ \text{W};\ 1.1\times10^{15}\ \text{W}.$$