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<u>Problem 41.39 (RHK)</u>

A plane electromagnetic wave, with wavelength 3.18 m, *travels in a free space in the* $+x$ *direction with its electric vector* $\vec{E}$, *of amplitude* $288\ \text{V m}^{-1}$, *directed along the* y *axis. We have to find* (a) *the frequency of the wave*; (b) *the direction and amplitude of the magnetic field associated with the wave*; (c) *If* $E = E_m \sin(kx - \omega t)$, *the values of* k *and* ω; (d) *the intensity of the wave*; *and* (e) *assuming that the wave falls upon a perfectly absorbing sheet of area* $1.85\ \text{m}^2$, *the rate with which the momentum is being delivered to the sheet and the radiation pressure exerted on the sheet*.

<u>Solution:</u>

(a)

Wavelength of the plane electromagnetic wave

$\lambda = 3.18\ \text{m}$.

The frequency of the wave will be

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{3.18}\ \text{Hz} = 9.43\times10^7\ \text{Hz} = 94.3\ \text{MHz}.$$

(b)

The amplitude of the electric field is

$E_m = 288$ V m^{-1}.

As the electric and magnetic fields for a plane wave are related as

$E_m = cB_m$,

the amplitude of the magnetic field will be

$$B_m = \frac{288}{3 \times 10^8} \text{ T} = 960 \text{ nT}.$$

As the electric field is directed along the y axis, we have

$$\vec{E} = \left|\vec{E}\right| \hat{j}.$$

As the wave is travelling in the $+x$ direction, the direction of the Poynting vector

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B},$$

will be $\hat{i}$. That is

$$\vec{S} = \left|\vec{S}\right| \hat{i}.$$

From the property of the cross product we note that the magnetic field will be in the z direction. That is

$$\vec{B} = \left|\vec{B}\right| \hat{k}.$$

(c)

We recall

$$k = \frac{2\pi}{\lambda} \text{ and } \omega = 2\pi\nu .$$

Therefore,

$$k = \frac{2\pi}{3.18} \text{ m}^{-1} = 1.976 \text{ m}^{-1},$$

and

$$\omega = 2\pi\nu = 2\pi \times 9.434 \times 10^{7} \text{ rad s}^{-1}$$
$$= 5.925 \times 10^{8} \text{ rad s}^{-1}.$$

(d)

Intensity of the wave

$$I = \bar{S} = \frac{1}{2\mu_0 c} E_m^2 = \frac{(288)^2}{2 \times 4 \times \pi \times 10^{-7} \times 3 \times 10^{8}} \text{ W m}^{-2}$$
$$= 110 \text{ W m}^{-2}.$$

(e)

When the wave falls normally upon a perfectly absorbing sheet of area 1.85 m^2, the rate at which momentum will be delivered to the sheet will be

$$F_{rad} = \frac{I}{c} \times A = \frac{110 \times 1.85}{3 \times 10^{8}} \text{ N} = 678 \text{ nN}.$$

The radiation pressure exerted on the sheet will be

$$\frac{F_{rad}}{A} = \frac{I}{c} = \frac{110}{3\times10^{8}} \text{ Pa} = 367 \text{ nPa.}$$