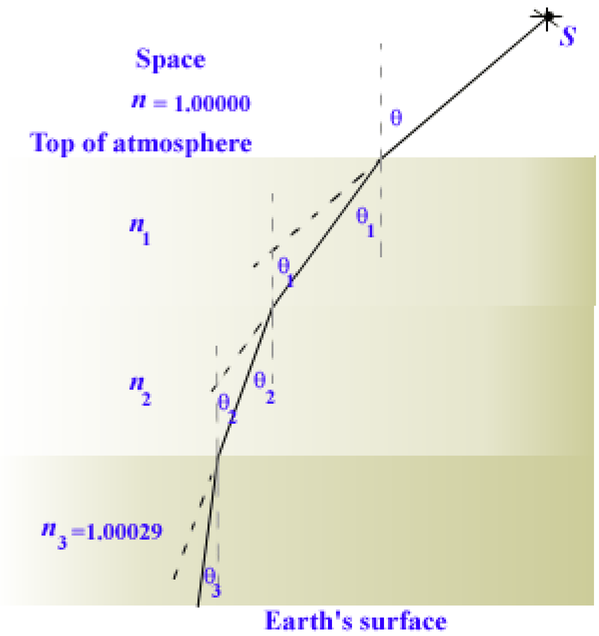
594.

Problem 43.20 (RHK)

The index of refraction of the Earth's atmosphere decreases monotonically with height from its surface value (*about* 1.00029) *to the value in space* (*about* 1.000000 *at the top of the atmosphere*. *This continuous* (*or graded*) *variation can be approximated by considering the atmosphere to be composed of three* (*or more*) *plane parallel layers in each of which the index of refraction is constant. Thus, in the figure,* $n_3 > n_2 > n_1 > 1.00000$. *We will consider a ray of light from a star S that strikes the top of the atmosphere at an angle* θ *with the vertical.* (a) *We have to show that the apparent direction* θ_3 *of the star with the vertical as seen by an observer at the Earth's surface is* obtained from

$$\sin\theta_3 = \frac{1}{n_3}\sin\theta.$$

(b) *We have to calculate the shift in position of a star observed to be* 50^0 *from the vertical*.



Solution:

(a)

We will use Snell's law successively for answering this problem.

The angle of refraction in the layer with refractive index n_1 of the ray from the star incident on the top of the atmosphere at incident angle θ is given by the Snell's law

$$n_1 \sin\theta_1 = \sin\theta .$$

The angle of refraction θ_2 of the ray incident on top of the layer with refractive index n_2 at angle θ_1 is given by the Snell's law

$n_2 \sin\theta_2 = n_1 \sin\theta_1.$

The angle of refraction θ_3 of the ray incident on top of the layer with refractive index n_3 at angle θ_2 is given by the Snell's law

$n_3 \sin\theta_3 = n_2 \sin\theta_2.$

Combining these relations, we note that

$n_3 \sin\theta_3 = \sin\theta,$

or

$$\sin\theta_3 = \frac{1}{n_3}\sin\theta.$$

(b)

We have to calculate the shift in position of a star observed to be 50^0 from the vertical.

For

$\theta = 50^0,$

and

$n_3 = 1.00029,$

$$\sin\theta_3 = \frac{\sin 50^0}{1.00029} = \frac{0.7660444}{1.00029} = 0.765822,$$

and

$$\theta_3 = \sin^{-1}(0.765822) = 49.98020.$$

And

$$\theta\text{-}\theta_3 = 0.01979^0.$$