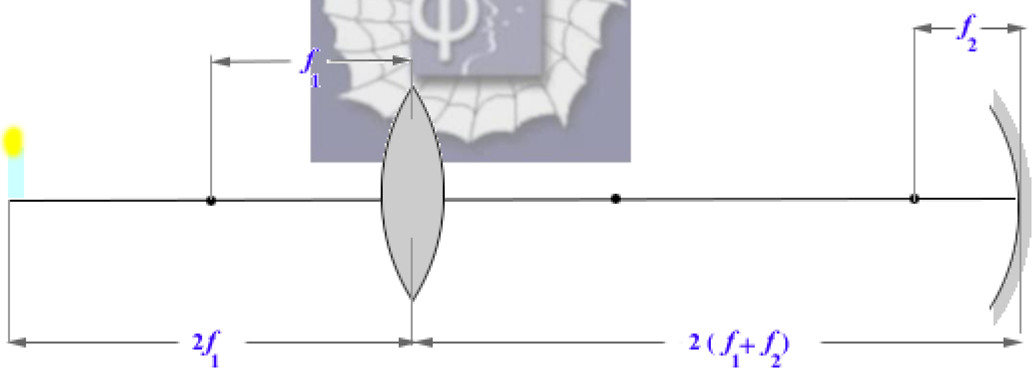
622.

Problem 44.30 (RHK)

An upright object is placed a distance in front of a converging lens equal to twice the focal length f_1 *of the lens. On the other side of the lens is a converging mirror of focal length* f_2 *separated from the lens by a distance* $2(f_1 + f_2)$; *see the figure.* (a) *We have to find the location, nature, and relative size of the image, as seen by an eye looking toward the image through the lens.*



Solution:

We will solve this problem in three steps. We first find the image of a candle placed at a distance $2f_1$ from a converging lens of focal length f_1. Let the image be formed at a distance i_1 from the lens. We use the thin lens equation for finding f_1. We have

$$\frac{1}{2f_1}+\frac{1}{i_1}=\frac{1}{f_1},$$

$$\therefore\ i_1=2f_1.$$

The first image formed by the converging lens will be real and inverted and will be at a distance $2f_1$ to the right of the lens. The distance of this image from the concave mirror, which is at a distance $2(f_1+f_2)$ from the converging lens, will therefore be $2f_2$. We now use the spherical mirror formula for locating the image formed by the concave mirror of the real image of the candle formed by the converging lens. We have

$$\frac{1}{2f_2}+\frac{1}{i_2}=\frac{1}{f_2},$$

or

$$i_2=2f_2.$$

This image will therefore be real and it will be at a distance $2f_2$ from the concave mirror.

As we are looking at the final image through the lens, we next calculate the image formed by the converging lens of the image formed by the concave mirror. As the object is real, its distance from the converging lens will be

$$o_3=(2f_1+2f_2-2f_2)=2f_1.$$

The final image will be at a distance i_3 from the converging lens.

$$\frac{1}{2f_1}+\frac{1}{i_3}=\frac{1}{f_1},$$

or

$$i_3 = 2f_1.$$

Therefore, the final image of the candle as seen through the lens will be real and at the location of the candle.

We now work out the lateral magnification. It will be

$$\begin{aligned} m = m_1 m_2 m_3 &= \left(-\frac{i_1}{o_1}\right)\times\left(-\frac{i_2}{o_2}\right)\times\left(-\frac{i_3}{o_3}\right) \\ &= \left(-\frac{2f_1}{2f_1}\right)\times\left(-\frac{2f_2}{2f_2}\right)\times\left(-\frac{2f_1}{2f_1}\right) \\ &= -1. \end{aligned}$$

Therefore, we find that the final image will be at the location of the candle, unchanged in size but will be inverted.