624.

Problem 44.33 (RHK)

Two thin lenses of focal length f_1 and f_2 are in contact. We have to show that they are equivalent to a single thin lens with a focal length given by

$$f = \frac{f_1 f_2}{f_1 + f_2} \ .$$

Solution:

Let us consider a beam of light parallel to the common axis of the two lenses, which is incident from the left on the lens of focal length f_1. The first lens of focal length f_1 will form a real image at its focal point, which is at a distance f_1 to the right of it. But to the second lens the rays will appear to come from a virtual object, as rays bent by the first lens enter it as though they are converging to a point to the right of the lens. Therefore, the object distance o_2 for the second lens will be $-f_1$. It will form an image at a distance i_2 from it. We find i_2 using the thin lens formula. We have

$$-\frac{1}{f_1}+\frac{1}{i_2}=\frac{1}{f_2},$$

or

$$i_2=\frac{f_1 f_2}{f_1+f_2}\ .$$

The image distance is positive. Therefore, the final image is real. As an incident parallel beam is focussed by the two lens combination at a distance $f=\frac{f_1 f_2}{f_1+f_2}$ from them, the two thin lenses in contact with each other are equivalent to a thin lens of focal length $f=\frac{f_1 f_2}{f_1+f_2}$.