707.

Problem 49.13 (RHK)

A convex lens 3.8 cm *in diameter and focal length* 26 cm *produces an image of the Sun on a thin black screen the same size as the image. We have to find the highest temperature to which the screen can be raised. The effective temperature of the Sun is* $5800\ \mathrm{K}$.

Solution:

For answering this problem we will make use of the following basic data of the Sun:

Mean radius of the Sun, $R_{sun} = 6.96\times 10^{8}\ \mathrm{m}$;

Mean orbital radius of the Earth about the Sun,

$R_{sun-earth} = 1.50\times 10^{11}\ \mathrm{m}$.

The size of the image of the Sun formed by lens of focal length 26 cm will be given by the relation

$$\frac{r_{image}}{R_{sun}} = \left|\frac{i}{o}\right|.$$

$$\therefore\ r_{image} = \frac{0.26}{1.50\times 10^{11}}\times 6.96\times 10^{8}\ \mathrm{m} = 1.21\ \mathrm{mm}.$$

The amount of energy falling per second on the thin black screen, which is of the same size as the screen, will be equal to the solar energy falling per second on the lens.

We calculate the energy radiated per second by the Sun. It is

$$
\begin{aligned}
E_{sun} &= \sigma T_{sun}^{4} \times \left(4\pi R_{sun}^{2}\right) \\
&= 5.67 \times 10^{-8} \times \left(5800\right)^{4} \times 4\pi \times \left(6.96 \times 10^{8}\right)^{2} \text{ W} \\
&= 3.90 \times 10^{26} \text{ W.}
\end{aligned}
$$

The energy from the Sun received at Earth per square meter and per second is

$$
I = \frac{E_{sun}}{4\pi R_{sun-earth}^{2}} = \frac{3.90 \times 10^{26}}{4\pi \times \left(1.50 \times 10^{11}\right)^{2}} \text{ W m}^{-2}
$$

$$
= 1380 \text{ W m}^{-2}.
$$

The amount of solar energy that falls on a lens 3.8 cm in diameter per second will be

$$
\begin{aligned}
E_{lens} &= \frac{\pi}{4} \times \left(3.8 \times 10^{-2}\right)^{2} \times 1380 \text{ W} \\
&= 1.564 \text{ W.}
\end{aligned}
$$

The highest temperature to which the black screen can be raised by focussing solar energy through the lens will be determined by E_{lens}. The screen acts as a black body at temperature T_{screen}, and at equilibrium the energy emitted per second by the screen will be equal to the energy received by it per second. Therefore,

$$\sigma T_{screen}^4 \times \pi \times r_{image}^2 = \mathsf{E}_{lens},$$

or

$$T_{screen}^4 = \frac{1.564}{5.67\times10^{-8}\times\left(1.21\times10^{-3}\right)^2}\ \mathrm{K}^4,$$

$$= 5.99\times10^{12}\ \mathrm{K}^4$$

and

$$T_{screen} = 1.565\times10^{3}\ \mathrm{K} = 1565\ \mathrm{K}.$$