731.

Problem 50.19 (RHK)

A beam of low-energy neutrons emerges from a reactor and is diffracted from a crystal. The kinetic energy of the neutrons is contained in a band of width ΔK centred on the kinetic energy K. We have to show that the angles for a given order of diffraction are spread over a range $\Delta\theta$ given in degrees by

$$\Delta\theta = \left(\frac{90}{\pi}\right)(\tan\theta)\frac{\Delta K}{K},$$

Where θ is the diffraction angle for a neutron with kinetic energy K.

Solution:

The de Broglie wavelength of a neutron of kinetic energy K is given by the expression

$$\lambda = \frac{h}{\sqrt{2m_{neutron}K}},$$

as momentum of a neutron of mass $m_{neutron}$ is

$$p = \sqrt{2m_{neutron}K}.$$

Therefore, as the spread of kinetic energy is ΔK centred on the kinetic energy K the corresponding spread of de Broglie wavelengths will be

$$\Delta\lambda = \frac{1}{2}\frac{\lambda}{K}\Delta K$$.

This is obtained by differentiating the de Broglie wavelength with respect to the variable *K*.

If the grating separation is d , the grating equation for diffraction maximum of order m at diffraction angle θ will be

$d\sin\theta = m\lambda$.

Therefore, the spread of diffraction angle if the spread of wavelength is $\Delta\lambda$ will be

$$\Delta\theta = \frac{m\Delta\lambda}{d\cos\theta} \text{ rad,}$$

or

$$\Delta\theta = \frac{m}{d\cos\theta}\left(\frac{1}{2}\frac{\lambda}{K}\right)\Delta K \text{ rad,}$$

$$= \frac{1}{2}\frac{\tan\theta}{K}\Delta K \text{ rad}$$

$$= \left(\frac{180}{\pi}\right)\frac{1}{2}\frac{\tan\theta}{K}\Delta K \text{ degrees}$$

$$= \left(\frac{90}{\pi}\right)(\tan\theta)\frac{\Delta K}{K} \text{ degrees.}$$