**787.**

**Problem 52.35 (RHK)**

*The use of lasers for defence against ballistic missiles is being studied. A laser beam of intensity* $120\ \text{MW m}^{-2}$ *would probably burn into and destroy a hardened* (*nonspinning*) *missile in about* 1 s. (a) *Assuming that the laser has a power output of* 5.30 MW, *a wavelength of* $2.95\ \mu\text{m}$, *and a beam diameter of* 3.72 m (*a very powerful laser indeed*), *we have to answer whether it would destroy a missile at a distance of* 3000 km. (b) *If the wavelength could be changed, we have to find the wavelength that would work.* (c) *If the wavelength of the laser could not be changed, we have to find the destructive range of the laser in* (a). *We may use the equation for the central diffraction disk and take the focal length to be the distance to the target.*

**Solution:**

(a)

We know that the radius of the central diffraction disk is given by

$$R = \frac{1.22 f \lambda}{d}.$$

We also know that the central disk contains about 84% of the incident power. In the equation given above for the radius of the central disk, we will use for the focal length $f$ the distance of the target from the laser lens. The wavelength of the laser is $2.95\ \mu\text{m}$ and the beam diameter is 3.72 m. Therefore,

$$R = \frac{1.22 \times 3 \times 10^{6} \times 2.95 \times 10^{-6}}{3.72}\ \text{m}$$

$$= 2.90\ \text{m}\ .$$

The output power of the laser is 5.30 MW. About 84% of the incident power is contained in the central disk. Therefore, the power flux of the laser at the target will be

$$= \frac{0.84 \times 5.30}{\pi \times \left(2.90\right)^{2}}\ \text{MW m}^{-2}$$

$$= 1.68 \times 10^{-1}\ \text{MW m}^{-2}.$$

It is much less than 120 MW m$^{-2}$ required for burning the target.

Let us find the radius of the central diffraction disk at the target, $R'$, for which the power flux at the target will be 120 MW m$^{-2}$.

$$120 \text{ MW m}^{-2} = \frac{0.84 \times 5.30}{\pi \times (R')^2} \text{ MW m}^{-2},$$

or

$$R' = \left( \frac{0.84 \times 5.30}{120 \times \pi} \right)^{1/2} \text{ m} = 1.086 \times 10^{-1} \text{ m}.$$

(b)

We will calculate next the wavelength $\lambda'$ of the laser beam that will produce a central diffraction disk of $1.086 \times 10^{-1}$ m at the target distance of 3000 km.

$$\lambda' = \frac{R' \times d}{1.22 f} = \frac{1.086 \times 10^{-1} \times 3.72}{1.22 \times 3 \times 10^{6}} \text{ m}$$

$$= 1.10 \times 10^{-7} \text{ m} = 0.110 \ \mu\text{m}.$$

(c)

If the wavelength of the laser beam could not be changed, the distance to the target at which the power flux will be 120 MW m$^{-2}$ can be calculated by the requirement that

$$\frac{1.22 f' \lambda}{d} = R',$$

or

$$f' = \frac{d \times R'}{1.22 \lambda} = \frac{3.72 \times 1.086 \times 10^{-1}}{1.22 \times 2.95 \times 10^{-6}} \text{ m}$$

$$= 1.122 \times 10^{5} \text{ m} = 112 \text{ km}.$$