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Problem 53.20 (RHK)

A neutron star can be analyzed by techniques similar to those used for ordinary metals. In this case the neutrons (*rather than electrons*) *obey the Fermi-Dirac probability function. We have to consider a neutron star of* 2.00 *solar masses with a radius of* 10.0 km *and calculate the Fermi energy of the neutrons*.

Solution:

Mass of a neutron, $m_n = 1.67\times10^{-27}\ \text{kg}$,

Mass of a neutron star of 2.00 solar masses,

$M = 2\times1.99\times10^{30}\ \text{kg}$,

Radius of the neutron star, $R = 10.0\times10^{3}\ \text{m}$.

Number of neutrons in the neutron star,

$$N = \frac{2\times1.99\times10^{30}}{1.67\times10^{-27}} = 2.383\times10^{57}.$$

From the above data we note that the number of neutron per cubic meter in the neutron star,

$$n = \frac{N}{\left(4\pi R^3/3\right)} = \frac{2.383\times10^{57}}{\left(4\pi\left(10^4\right)^3/3\right)} \text{ neutrons per m}^3$$

$$= 5.688\times10^{44} \text{ neutrons per m}^3.$$

At absolute zero Fermi energy is the highest energy of the occupied state. It is given by the relation

$$E_F = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3}.$$

Therefore, the Fermi energy of the neutrons in the neutron star of 2 solar masses and radius of 10 km will be

$$E_F = \frac{\left(6.63\times10^{-34}\right)^2}{8\times1.67\times10^{-27}}\times\left(\frac{3\times5.688\times10^{44}}{\pi}\right)^{2/3} \text{ J}$$

$$= 21.9\times10^{-12} \text{ J}$$

$$= 21.9\times10^{-12}\times6.242\times10^{12} \text{ MeV}$$

$$= 136.7 \text{ MeV}.$$