849.

Problem 54.66 (RHK)

A rock is found to contain 4.20 mg *of* $^{238}\mathrm{U}$ *and* 2.00 mg *of* $^{206}\mathrm{Pb}$. *We may assume that the rock contained no lead at formation, all the lead now present has arisen from the decay of the uranium. We have to find the age of the rock. The half-life of Uranium is* 4.47×10^{9} y.

Solution:

A rock is found to contain 4.20 mg of $^{238}\mathrm{U}$ and 2.00 mg of $^{206}\mathrm{Pb}$. Let the age of the rock be t y. We calculate the number of $^{238}\mathrm{U}$ in 4.20 mg.

$$N_{^{238}\mathrm{U}}(t)=\frac{6.02\times10^{23}\times4.20\times10^{-3}\ \mathrm{g}}{238\ \mathrm{g}}=1.06\times10^{19}.$$

And, the number of $^{206}\mathrm{Pb}$ in 2.00 mg will be

$$N_{^{206}\mathrm{Pb}}\left(t\right)=\frac{6.02\times10^{23}\times2.0\times10^{-3}\ \mathrm{g}}{206\ \mathrm{g}}=5.84\times10^{18}.$$

Let the number of $^{238}\mathrm{U}$ in the rock when it was formed that is at $t=0$ be $N_{^{238}\mathrm{U}}(t=0)$.

The half-life of $^{238}\mathrm{U}$ is 4.47×10^{9} y.

An equivalent form of the radioactive decay law is

$$N(t) = N(0)\left(\frac{1}{2}\right)^{t/t_{1/2}},$$

or

$$2^{t/t_{1/2}} = \frac{N(0)}{N(t)}.$$

We note that

$$N_{^{238}\mathrm{U}}(0) - N_{^{238}\mathrm{U}}(t) = N_{^{206}\mathrm{Pb}}(t) = 5.84 \times 10^{18}.$$

Therefore,

$$\frac{N_{^{238}\mathrm{U}}(0) - N_{^{238}\mathrm{U}}(t) = N_{^{206}\mathrm{Pb}}(t)}{N_{^{238}\mathrm{U}}(t)} = \frac{5.84 \times 10^{18}}{1.06 \times 10^{19}}$$

$$= 0.551.$$

And,

$$2^{t/t_{1/2}} - 1 = 0.551,$$

or

$$2^{t/t_{1/2}} = 1.551$$

or

$$\frac{t}{t_{1/2}} = \frac{\ln 1.551}{\ln 2} = 0.633.$$

And,

$$t = 0.633 \times 4.47 \times 10^{9} \text{ y} = 2.83 \times 10^{9} \text{ y}.$$