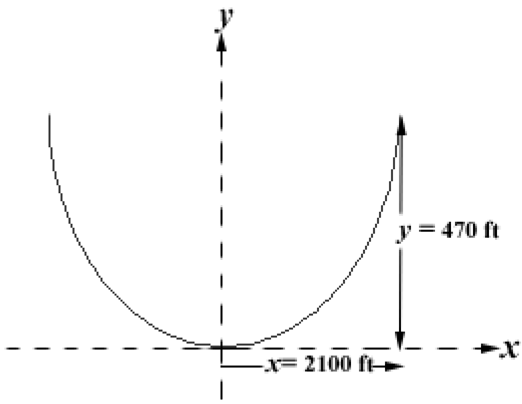
198.

Problem 22.49 (RHK)

The distance between the towers of the main span of Golden Gate Bridge near San Francisco is 4200 ft. *The sag of the cable halfway between the towers at* 50^{0}F *is* 470 ft. *Assuming that the coefficient of linear expansion for the cable is* $\alpha = 6.5\times10^{-6}\,\text{F}^{0}$, *we have to compute the change in length of the cable for a temperature change from* 10 to 90^{0}F. *We can assume that there is no bending or separation of the towers and that the shape of the cable is parabolic.*

Solution:



Let the equation of the parabola be

$$y = \frac{x^2}{2a}.$$

We will calculate the length of the parabolic curve from the coordinate (-x,y) to (0,0) to (x,y). We call this length l. It is given by the integral

$$l = 2 \times \int_0^x dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{2}{a} \times \int_0^x dx \left(a^2 + x^2\right)^{1/2}$$

$$= \frac{1}{a}\left[x\sqrt{x^2 + a^2} + a^2 \ln\left(\frac{x + \sqrt{x^2 + a^2}}{a}\right)\right].$$

We will fix the function describing the parabola by calculating the constant *a* using the data that the point $x = 2100$ and $y = 470$ is on the parabolic curve. Therefore,

$$a = \frac{(2100 \text{ ft})^2}{2 \times 470 \text{ ft}} = 4691.5 \text{ ft}.$$

We can now calculate the length of the cable joining the towers at temperature 50^0F. Substituting $x = 2100$ ft and $a = 4691.5$ ft in the expression for *l*, we find

$$l\left(50^0\text{F}\right) = 4336.3 \text{ ft.}$$

Using the value of the coefficient of linear expansion for the cable, $\alpha = 6.5 \times 10^{-6} / {}^0\text{F}$, we estimate the length of the cable at 10^0F. We find

$$
\begin{aligned}
l\left(10^{0}\mathrm{F}\right) &= l\left(50^{0}\mathrm{F}\right) - \Delta l \\
&= 4336.3\ \mathrm{ft} - 4336.3 \times 6.5 \times 10^{-6} \times 40\ \mathrm{ft} \\
&= 4335.2\ \mathrm{ft}.
\end{aligned}
$$

Therefore, the change in length of the cable when the temperature changes from $10^{0}\mathrm{F}$ to $90^{0}\mathrm{F}$ will be

$$
\begin{aligned}
\Delta l &= 4335.2 \times 6.5 \times 10^{-6} \times 80\ \mathrm{ft} \\
&= 2.25\ \mathrm{ft}.
\end{aligned}
$$