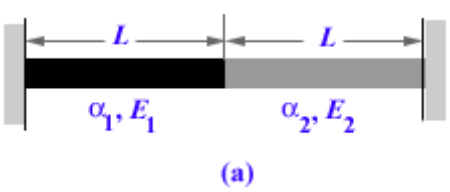
204.

Problem 22.46 (RHK)

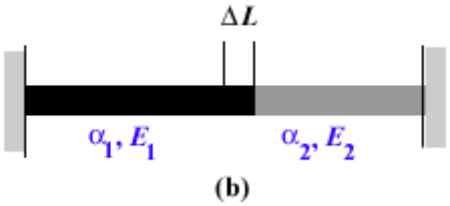
Two rods of different materials bur having the same lengths L and cross-sectional area A are arranged end-to-end between fixed rigid supports. The temperature is T and there is no initial stress. The rods are heated, so that their temperature increases by ΔT. (a) *We have to show that the rod interface is displaced upon heating by an amount given by*

$$\Delta L = \left(\frac{\alpha_1 E_1 - \alpha_2 E_2}{E_1 + E_2} \right) L \Delta T,$$

where α_1 *and* α_2 *are the coefficients of linear expansion and* E_1, E_2 *are Young's moduli of the materials. We may ignore changes in the cross-sectional areas.* (b) *We have to find the stress at the interface after heating.*



(a)



(b)

Solution:

As the rods gets heated they undergo linear thermal expansion, but as there ends are clamped with rigid supports stress develops at the joint of the two rods. Let *S* be the stress at the interface when the temperature has increased by ΔT. The change in length of each rod is the combined effect of thermal expansion and elastic change due to stress.

As shown in the diagram (b) the length of the rod 1 after heating becomes $L+\Delta L$ and that of the rod 2 becomes $L-\Delta L$.

Because of the stress length of rod 1 will decrease by SL/E_1 and that of the rod 2 by SL/E_2. And because of change in temperature the length of the rod 1 will increase by $\alpha_1 L\Delta T$ and the length of the rod 2 will increase by $\alpha_2 L\Delta T$. The resultant changes in lengths are

$$\Delta L = -\frac{SL}{E_1} + \alpha_1 L \Delta T,$$

and

$$-\Delta L = -\frac{SL}{E_2} + \alpha_2 L \Delta T$$

We have two linear equations and two unknowns, ΔL and S. By solving the algebraic equations, we find

$$\Delta L = \left(\frac{\alpha_1 E_1 - \alpha_2 E_2}{E_1 + E_2} \right) L \Delta T,$$

and

$$S = \frac{E_1 E_2}{E_1 + E_2} \times (\alpha_1 + \alpha_2) \times \Delta T.$$