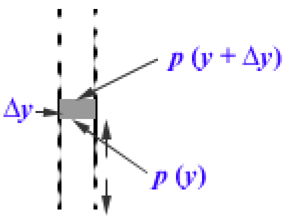
215.

<u>Problem 23.6 (RHK)</u>

We will show that the variation in pressure in the Earth's atmosphere, assumed to be at a uniform temperature, is given by $p = p_0 e^{-Mgy/RT}$*, where M is the molar mass of the air. We will also show that* n_y *the number of molecules per unit volume at the height y above the sea level varies as* $n_y = n_0 e^{-Mgy/RT}$.

<u>Solution:</u>



Consider a vertical column of unit cross-sectional area. We set up the equilibrium condition of an air column of width Δy and unit cross-sectional area at height y from the surface of the Earth. Its weight $\rho(y)g\Delta y$ will be balanced by the pressure difference at its top and bottom faces. That is

$$p(y) = p(y + \Delta y) + \rho(y)g\Delta y.$$

This gives a first order differential equation for the function $p(y)$,

$$\frac{dp(y)}{dy} = -\rho(y)g.$$

Assuming that the air column is in thermodynamic equilibrium and its equation of state is described by the ideal gas equation,

$$p(y) = \frac{\rho(y)RT}{M},$$

where M is the molar mass of air. We now have a differential equation giving the variation of air density with height y. It is

$$\frac{d\rho}{dy} = -\frac{Mg}{RT}\rho.$$

Solution of this differential equation is

$$\rho(y) = \rho_0 e^{-gMy/RT},$$

where ρ_0 is the density of air at the surface of the Earth. Under the assumption that the temperature of the air column does not vary with height y, we find the following equation for pressure variation:

$$p = p_0 e^{-Mgy/RT}.$$

An alternative form of the ideal gas equation is

$$pV = Nkt,$$

where N is the total number of molecules in volume V.

Or

$$p = nkT,$$

where n is the number of molecules per unit volume. We thus find $n(y)$ varies as

$$n_y = n_0 e^{-Mgy/RT}.$$