**262.**

## Problem 26.41 (RHK)

*One mole of a monatomic ideal gas is taken from an initial state of pressure $p_0$ and volume $V_0$ to a final state of pressure $2p_0$ by two different processes. (1) It expands isothermally until its volume is doubled, and then its pressure is increased at constant volume to the final state. (2) It is compressed isothermally until its pressure is doubled, and then its volume is increased at constant pressure to the final state. We have to show the path of each process on a pV diagram. For each process we have to calculate in terms of $p_0$ and $V_0$: (a) the heat absorbed by the gas in each part of the process; (b) the work done on the gas in each part of the process; (c) the change in the internal energy of the gas, $E_{in,f} - E_{in,i}$; and (d) the change in entropy of the gas, $S_f - S_i$.*
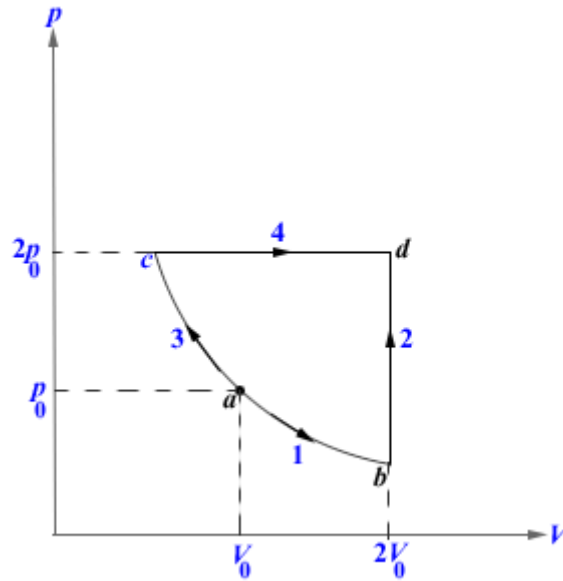
## Solution:

We are considering one mole of a monatomic ideal gas as it undergoes thermodynamic processes as shown in the diagram. Its initial state is at the point $a$ in the diagram. At $a$ pressure of the gas is $p_0$ and its volume is $V_0$. Its temperature at a will therefore be

$$T_0 = \frac{p_0 V_0}{R}.$$

(1)

As the gas undergoes isothermal expansion at temperature $T_0$ from volume $V_0$ to $2V_0$ from $a$ to $b$ along the path 1, the work done on the gas will be

$$W_1 = -\int_{V_0}^{2V_0} p\, dV = -\int_{V_0}^{2V_0} \frac{RT_0}{V}\, dV = -RT_0 \ln 2 = -p_0 V_0 \ln 2.$$

As the process is isothermal there is no change in the internal energy of the gas, therefore, the heat absorbed by the gas, $Q_1$ , during the process 1 will be

$$Q_1 = -|W_1| = p_0 V_0 \ln 2.$$

The change in entropy

$$S_b - S_a = \frac{Q_1}{T_0} = \frac{RT_0 \ln 2}{T_0} = R \ln 2.$$

(2)

The process b to d takes place at constant volume. Therefore, no work is done on the gas during this process. Therefore, the heat absorbed by the gas during 2 will be

$$Q_2 = C_V \left( T_d - T_b \right)$$
$$= \frac{3R}{2} \left( \frac{4 p_0 V_0}{R} - \frac{p_0 V_0}{R} \right) = \frac{9}{2} p_0 V_0.$$

We have used that for a monatomic gas

$$C_V = \frac{3}{2} R,$$

and at b

$$p = \frac{p_0}{2}, \ V = 2 V_0, \ T = \frac{pV}{R} = \frac{p_0 V_0}{R}.$$

And at d

$$p = 2p_0, \quad V = 2V_0, \quad \text{and} \quad T = \frac{4p_0V_0}{R}.$$

And

$$E_{in,d} - E_{in,b} = E_{in,d} - E_{in,a} = \frac{9}{2}p_0V_0.$$

The change in entropy of the gas during the process 2 will be

$$S_d - S_b = \int\limits_{T_0}^{4T_0} \frac{C_V dT}{T} = \frac{3}{2}R\ln\left(\frac{4T_0}{T_0}\right) = 3R\ln 2.$$

The change in entropy from $a$ to $d$ in the combined processes 1 and 2 will be

$$S_d - S_a = S_d - S_b + S_b - S_a = 3R\ln 2 + R\ln 2 = 4R\ln 2.$$

(3)

We consider part 3 process. It is an isothermal compression at temperature $T_0$ from volume $V_0$ to $V_0/2$.

Therefore, the work done on the gas

$$W_3 = -RT_0\ln\left(\frac{V_0/2}{V_0}\right) = RT_0\ln 2,$$

and

$$Q_3 = -|W_3| = -p_0V_0\ln 2.$$

Therefore,

$$S - S_a = \frac{Q_3}{T_0} = -R\ln 2.$$

(4)

In the process 4 the gas is subjected to an isobaric expansion from volume $V_0/2$ to $2V_0$ at pressure $2p_0$. Therefore, the work done on the gas during the path 4 will be

$$W_4 = -2p_0\left(2V_0 - V_0/2\right) = -3p_0V_0.$$

And the heat absorbed by the gas during 4 will be

$$Q_4 = \int_{T_0}^{4T_0} C_p dT = \frac{5}{2}R\left(4T_0 - T_0\right) = \frac{15}{2}RT_0 = \frac{15}{2}p_0V_0.$$

We have used that for an ideal monatomic gas

$$C_p = \frac{5}{2}R.$$

And

$$E_{in,d} - E_{in,c} = E_{in,d} - E_{in,a} = W_4 + Q_4 = \frac{9}{2}p_0V_0.$$

The change in entropy

$$S_d - S_c = \int_{T_0}^{4T_0} C_p dT = \frac{5}{2}R\ln 4 = 5R\ln 2.$$

Therefore,

$$S_d - S_a = S_d - S_c + S_c - S_a = 5R\ln 2 + \left(-R\ln 2\right) = 4R\ln 2.$$