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Problem 10.14 (Zemansky)

Two identical bodies of constant heat capacity are at the same initial temperature T_i. *A refrigerator operates between these two bodies until one body is cooled to temperature* T_2. *If the bodies remain at constant pressure and undergo no change of phase, we have to show that the minimum amount of work needed to do this is*

$$W(\min) = mc_p\left(\frac{T_i^2}{T_2} + T_2 - 2T_i\right).$$

Solution:

We are given that we have two identical bodies each of mass *m* at temperature T_i. A refrigerator operates between these two bodies until one body is cooled to temperature T_2. Let the amount of work done on the refrigerator in cooling of body 2 from temperature T_i to T_2 be *W*. The quantity of heat extracted out of body 2 will be

$$Q = mc_p \left(T_i - T_2\right).$$

And the amount of heat delivered by the refrigerator to the body 1 will be

$$Q' = Q + W.$$

Temperature of body 1 will have got raised from T_i to T' such that

$$mc_p \left(T' - T_i\right) = Q + W$$

$$= mc_p \left(T_i - T_2\right) + W.$$

From this equation, we find

$$T' = 2T_i - T_2 + \frac{W}{mc_p}.$$

We next calculate the change in entropy of the body 1 and that of the body 2.

$$\Delta S_1 = mc_p \ln\left(\frac{T'}{T_1}\right) = mc_p \ln\left(\frac{2T_i - T_2 + \frac{W}{mc_p}}{T_i}\right).$$

And

$$\Delta S_2 = mc_p \ln\left(\frac{T_2}{T_i}\right).$$

The total change of entropy in the process will be

$$\Delta S = \Delta S_1 + \Delta S_2 = mc_p \ln\left(\frac{\left(2T_i - T_2 + \frac{W}{mc_p}\right)\times T_2}{T_i^2}\right).$$

From the entropy principle

$\Delta S \geq 0.$

Minimum amount of work will be done on the refrigerator in the process for which

$\Delta S = 0.$

That is

$$\frac{\left(2T_i - T_2 + \frac{W(\min)}{mc_p}\right)\times T_2}{T_i^2} = 1,$$

or

$$W(\min) = mc_p\left(\frac{T_i^2}{T_2} + T_2 - 2T_i\right).$$